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Approximate reasoning from sampled data

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Abstract. The subject of this paper is a mathematical framework that provides a basis and a tool for reasoning about uncertainty based on linguistic information.

1. Introduction

The semantics of certain classes of words and language expressions can be modeled by fuzzy sets which have proven to be a very useful tool in capturing information that is not certain and precise but rather uncertain or imprecise. The research was initiated by L. A. Zadeh in his early papers and since then, most of the applications of fuzzy sets have emphasized the presence of natural language, at least in a hidden form [2]. Fuzzy natural logic is a mathematical framework that focuses on the formalization of human reasoning processes expressed through natural language. It combines methods from fuzzy set theory and logic in order to describe and analyze reasoning with vague and imprecise information. This framework is organized into three main components: (i) theory of evaluative linguistic expressions, (ii) theory of fuzzy/linguistic *IF−THEN* rules and logical inference based on them and (iii) theory of fuzzy generalized quantifiers [see (11,13,15)].

The formulation and processing of fuzzy rules are performed within a fuzzy inference engine, which represents the central component of a fuzzy inference system. The main task of the inference engine is to evaluate the activated rules, combine the available fuzzy information, and derive appropriate conclusions from the given input data [13,15]. To achieve this, it relies on fundamental concepts of theory of fuzzy sets and systems, including fuzzy intersection, fuzzy union, and fuzzy implication operators. The inference engine extends the capabilities of fuzzy systems by providing a

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mechanism for approximative reasoning under uncertainty and transforming available information into meaningful conclusions [see (10,11,12)]. Through the evaluation and combination of fuzzy rules, it is able to capture complex dependencies between input and output variables and to incorporate different sources of knowledge within a unified framework [1,3,9]. This ability to process both quantitative data and qualitative descriptions makes fuzzy inference engines suitable for applications where conventional mathematical models cannot adequately represent the complexity of real-world processes. The effectiveness and behavior of a fuzzy inference engine depend strongly on the characteristics of the considered problem, including the choice of linguistic variables, membership functions, rule base, and fuzzy operators [7]. Therefore, the choice of inference mechanisms and parameters depends on the specific application [for more, see (4,5,6,8)].

2. PERCEPTION-BASED LOGICAL DEDUCTION

Data collection represents any activity related to the direct collection of information from examples in practice, statistical research, or downloading data from administrative sources, as well as data collection by the method of constant monitoring and observation. In the scope, by the term data collection, we mean a set of vectors $\{(X_0^p, y_0^p) \mid p \in 1, \ldots, N\}$, where input $X_0^p \in U = [\alpha_1, \beta_1] \times .. \times [\alpha_n, \beta_n]$, for some n-dimensional cube U, and output $y_0^p \in V = [\alpha_y, \beta_y] \subset R$. In this way, the input-output data collection is given by Table 1.

TABLE 1. Input-output data collection.

Input	C_1	C_2	$\cdots$	C_i	$\cdots$	C_n	Output
X_0^1	x_{01}^1	x_{02}^1	$\cdots$	x_{0i}^1	$\cdots$	x_{0n}^1	y_0^1
X_0^2	x_{01}^2	x_{02}^2	$\cdots$	x_{0i}^2	$\cdots$	x_{0n}^2	y_0^2
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
X_0^p	x_{01}^p	x_{02}^p	$\cdots$	x_{0i}^p	$\cdots$	x_{0n}^p	y_0^p
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
X_0^N	x_{01}^N	x_{02}^N	$\cdots$	x_{0i}^N	$\cdots$	x_{0n}^N	y_0^N

We restrict our attention to multi-input-single-output case because any multi-output system can be decomposed into a set of single-output systems, each modeled independently using the same inputs.

Using methodology described in [8], each attribute C_i, $i \in \{1, 2, \ldots, n\}$ is assigned with a collection of quadruples $(a_i^j, b_i^j, c_i^j, d_i^j)$, where $a_i^j, b_i^j, c_i^j, d_i^j \in [\alpha_i, \beta_i]$, $j \in \{1, 2, \ldots, N_i\}$. It is well known that in a natural way, these

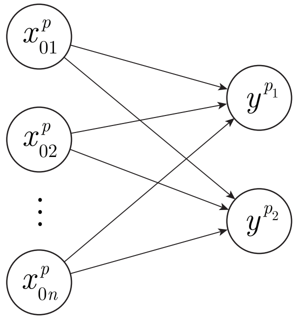

Figure 1. Multi-output-single-input system.

values induce the corresponding trapezoidal fuzzy sets

$$\mathcal{A}_i^j : [\alpha_i, \beta_i] \to [0,1], \qquad j = 1, 2, \ldots, N_i,$$

as follows:

$$\mathcal{A}_i^j(x) = \begin{cases} 0, & \alpha_i \le x \le a_i^j \ \text{ or } \ d_i^j \le x \le \beta_i, \\ \frac{x - a_i^j}{b_i^j - a_i^j}, & a_i^j \le x \le b_i^j, \\ 1, & b_i^j \le x \le c_i^j, \\ \frac{x - d_i^j}{c_i^j - d_i^j}, & c_i^j \le x \le d_i^j. \end{cases}$$

Let us note that it is for each $i = 1, 2, \ldots, n$, the set of fuzzy functions $\mathcal{A}_i^j$, $j = 1, 2, \ldots, N_i$, is complete in $[\alpha_i, \beta_i]$, i.e., for every $x_i \in [\alpha_i, \beta_i]$, there exists $\mathcal{A}_i^j$ such that $\mathcal{A}_i^j(x_i) \neq 0$.

In a similar way, output column $(y_0^1, y_0^2, \ldots, y_0^N)^T$ is assigned with quadruple (a_i, b_i, c_i, d_i) and trapezoidal fuzzy functions

$$\mathcal{B}^j : [\alpha_y, \beta_y] \to [0,1], \qquad j = 1, 2, \ldots, N_y,$$

defined by:

$$\mathcal{B}^j(x) = \begin{cases} 0, & \alpha_y \le x \le a^j \ \text{ or } \ d^j \le x \le \beta_y; \\ \frac{x - a^j}{b^j - a^j}, & a^j \le x \le b^j; \\ 1, & b^j \le x \le c^j; \\ \frac{x - d^j}{c^j - d^j}, & c^j \le x \le d^j. \end{cases}$$

Again, the assumption is that they are complete in $[\alpha_y, \beta_y]$.

Notice that the obtained fuzzy sets are trapezoidal, and in a natural way, they can be transformed into triangular fuzzy sets or singletons.

Now, we will define rules generated by input-output presented in Table 1. Namely, for every input-output (X_0^p, y_0^p), $p = 1, 2, \ldots, N$, and corresponding inputs and output we will determine the membership values

$$\mathcal{A}_i^j(x_{0i}^p), \qquad j = 1, 2, \ldots, N_i,$$

and the membership values

$$\mathcal{B}^l(y_0^p), \qquad l = 1, 2, \ldots, N_y.$$

Then for every input variable x_{0i}^p, $i = 1, 2, \ldots, n$, we will determine the fuzzy set in which x_{0i}^p has the largest membership value, that is, we will determine $\widetilde{\mathcal{A}}_i^j$ such that

$$\widetilde{\mathcal{A}}_i^j(x_{0i}^p) \geq \mathcal{A}_i^j(x_{0i}^p), \quad j = 1, 2, \ldots, N_i.$$

Similarly, we will determine $\widetilde{\mathcal{B}}^l$ such that

$$\widetilde{\mathcal{B}}^l(y_0^p) \geq \mathcal{B}^l(y_0^p), \quad 1 = 1, 2, \ldots, N_y.$$

From this we derive the fuzzy *if–then* rule

$$\text{(1)} \qquad \mathcal{R}_p: \quad \textit{If} \quad x_1 \textit{ is } \widetilde{\mathcal{A}}_1^j \quad \textit{and} \quad \cdots \quad \textit{and} \quad x_n \textit{ is } \widetilde{\mathcal{A}}_n^j \quad \textit{then} \quad y \textit{ is } \widetilde{\mathcal{B}}^l.$$

The part of *if–then* rule which follows *if* is called antecedent and the part which follows *then* is called consequent. If we observe from the surface level, the *if–then* rule is a conditional sentence of natural language. On the level of formal syntax, any rule, and consecutively the whole linguistic description, is assigned a certain formula of a chosen formal system. On the semantic level, that formula is assigned to a special set of functions over fuzzy relations, i.e. it is properly interpreted [10].

Since the number of input-output pairs is usually large, and for every pair one rule is generated, it is highly likely that there are conflicting rules, i.e., there are rules with the same *if* part and the different *then* part. In order to overcome this conflict, the degree is assigned to each rule previously generated. Only one rule from a conflicting group that has the maximum degree is chosen. That procedure resolves the conflict problem, but also reduces the number of rules.

In situations where one input parameter carries more information than another, weights are assigned directly to individual conditions within the rules. So, if the input-output pairs have different reliability, then we can assign a weight $w_j^p \in [0, 1]$, and the degree of the rule generated by a pair (X_0^p, y_0^p) can be defined by:

$$\text{(2)} \qquad D(\mathcal{R}_p)(X_0^p, y_0^p) = \prod_{i=1}^{n} w_i * \widetilde{\mathcal{A}}_i^j(x_{0i}^p) * \widetilde{\mathcal{B}}^l(y_0^p).$$

Weighting coefficients are numerical values assigned to a set of conditions to express their relative importance, priority, or impact compared to others in the same group.

In most analytical systems, weights ρ_i, $i = 1, 2, \ldots, n$ are normalized so that their total sum equals 1:

$$\text{(3)} \qquad \sum_{i=1}^{n} \rho_i = 1.$$

Let us form a matrix of ratio degrees between the weighted coefficients:

$$M = [m_{ik}]_{i,k=1}^{n} = \left[\frac{\rho_i}{\rho_k}\right]_{i,k=1}^{n} \tag{4}$$

By multiplying the elements in each row, we obtain the products $\pi_1, \pi_2, \ldots, \pi_n$, from which the average ratio degree can be calculated as follows:

$$w_i = \sqrt[n]{\pi_i},\ i = 1, 2, \ldots, n. \tag{5}$$

Theorem 1. *Coefficients* (5) *are normalized weighted coefficients for multiplicative models.*

Proof. Let ρ_i, $i = 1, 2, \ldots, n$ satisfy condition (3). Then for each $i = 1, 2, \ldots, n$, we have

$$w_i = \sqrt[n]{\pi_i} = \sqrt[n]{\prod_{k=1}^{n}\left(\frac{\rho_i}{\rho_k}\right)} = \sqrt[n]{\frac{\rho_i^n}{\prod_{k=1}^{n}\rho_k}} = \frac{\rho_i}{\sqrt[n]{\prod_{k=1}^{n}\rho_k}},$$

and consequently

$$\prod_{k=1}^{n} w_n = 1.$$

□

Finally, the fuzzy rule base consists of all rules that do not conflict with any other rules, and from each conflicting group we take the rule that has the maximum degree (where we assume that a group of conflicting rules consists of rules with the same *if* parts). On the basis of general knowledge and familiarity with the concrete situation, linguistic rules from human experts are often added to this set of rules.

3. Algorithm

Following the methodology introduced in Section 2, the fuzzy system is developed using the algorithm described below.

1. *Experts opinion*

   Experts' opinions on each attribute of the observed process are collected and analyzed in order to define the corresponding fuzzy functions within the proposed model. These expert assessments represent the basis for transforming qualitative knowledge and subjective judgments into mathematical representations suitable for fuzzy reasoning. By using the experts' evaluations, appropriate membership functions are constructed for each attribute/criterion, allowing the model to incorporate uncertainty, imprecision, and the inherent subjectivity present in the observed process.

2. *Fuzzy rules generated based on the available data*

   The fuzzy rules are generated based on the available input-output data, which represent the relationship between the system inputs and the corresponding outputs. These rules capture the underlying dependencies within the observed process and provide the basis for the inference mechanism of the fuzzy model.
3. *Calculation of the degrees of fulfillment of fuzzy rules*

   The activation degree of each fuzzy rule is calculated based on the membership degrees of the input variables. These values represent the extent to which each rule is satisfied for a given set of input data and are subsequently used in the fuzzy inference process.
4. *Fuzzy rule base*

   The fuzzy rule base represents the core component of the proposed fuzzy inference system and contains a set of fuzzy rules that describe the relationships between input variables and the corresponding output variable. The rule base is constructed by combining automatically generated rules from input-output data with expert knowledge expressed in the form of linguistic rules.

   The fuzzy rule base consists of the following groups of rules:

   - Non-conflicting rules generated in Step 2. These rules are directly included in the fuzzy rule base because each input combination is associated with a unique output conclusion. Since no contradiction exists among these rules, they can be applied without additional processing.

   - Selected rules from conflicting rule groups. Namely, during the rule generation process, several rules with identical IF parts but different THEN conclusions may be obtained. Such rules form a conflicting group because the same input conditions lead to different outputs. To resolve these conflicts, the rule with the highest degree of fulfillment is selected and included in the final fuzzy rule base. The degree of fulfillment represents the strength or reliability of a rule and indicates how well the available input-output data support that particular rule. Linguistic rules provided by human experts.

   - The fuzzy rule base is additionally enriched with linguistic rules obtained from experts' knowledge and experience. These rules represent explicit knowledge about the observed process that may not be fully captured by available numerical data. The inclusion of expert rules improves the interpretability of the fuzzy model and enables the representation of qualitative relationships, uncertainties, and process-specific characteristics.
5. *Construction of the Fuzzy System*

   In this step of the algorithm, the fuzzy system is constructed using the fuzzy rule base established in Step 4. The obtained rule base represents the knowledge structure of the system and defines the relationships between input variables and the corresponding output responses.

4. Example

In [8] model is presented using Mamdani's minimum implication and individual rule base inference is combined through union. Here we will present the method based on algebraic product for the t-norm. Such method is convenient for practical applications in multiplicative models.

Let us observe some sampled input-output data presented in Table 2, where input data is described with three attributes A_1, A_2, A_3 with different degrees of importance on outcome. This is represented by weights

$$\rho_1 = 0.5, \quad \rho_2 = 0.3, \quad \rho_3 = 0.2.$$

Using Theorem 1 these weights are modified for multiplicative method, so we obtain

$$w_1 = 1.61, \quad w_2 = 0.97, \quad w_3 = 0.64.$$

Using algorithm from Section 3, fuzzy rule base is constructed and all rules are presented in Table 3.

Table 2. Input-output data collection

Input	A_1	A_2	A_3	Output
X_0^1	2	3	2	2
X_0^2	3	2	1	1
X_0^3	9	7	6	5
X_0^4	5	4	1	1
X_0^5	8	6	5	5
X_0^6	2	6	1	4
X_0^7	4	2	2	1
X_0^8	3	7	5	3
X_0^9	1	4	3	2
X_0^{10}	9	7	6	1
X_0^{11}	1	4	3	3
X_0^{12}	8	6	5	4

TABLE 3. Rules

$\mathcal{R}_1$	IF x_1 in A_1^1 and x_2 in A_2^2 and x_3 in A_3^2 then y in B^2
$\mathcal{R}_2$	IF x_1 in A_1^1 and x_2 in A_2^2 and x_3 in A_3^2 then y in B^2
$\mathcal{R}_3$	IF x_1 in A_1^3 and x_2 in A_2^1 and x_3 in A_3^1 then y in B^1
$\mathcal{R}_4$	IF x_1 in A_1^2 and x_2 in A_2^2 and x_3 in A_3^2 then y in B^2
$\mathcal{R}_5$	IF x_1 in A_1^3 and x_2 in A_2^1 and x_3 in A_3^1 then y in B^1
$\mathcal{R}_6$	IF x_1 in A_1^1 and x_2 in A_2^1 and x_3 in A_3^2 then y in B^2
$\mathcal{R}_7$	IF x_1 in A_1^2 and x_2 in A_2^2 and x_3 in A_3^2 then y in B^2
$\mathcal{R}_8$	IF x_1 in A_1^1 and x_2 in A_2^1 and x_3 in A_3^1 then y in B^2
$\mathcal{R}_9$	IF x_1 in A_1^1 and x_2 in A_2^2 and x_3 in A_3^2 then y in B^2
$\mathcal{R}_{10}$	IF x_1 in A_1^3 and x_2 in A_2^1 and x_3 in A_3^1 then y in B^2
$\mathcal{R}_{11}$	IF x_1 in A_1^1 and x_2 in A_2^2 and x_3 in A_3^2 then y in B^2
$\mathcal{R}_{12}$	IF x_1 in A_1^3 and x_2 in A_2^1 and x_3 in A_3^1 then y in B^2

As it can be seen, Table 3 contains conflicting group of rules, so in each subgroup of conflicting rules we take the rule with highest degree.

TABLE 4. Reduced rules

$\mathcal{R}'_1$	IF x_1 in A_1^1 and x_2 in A_2^2 and x_3 in A_3^2 then y in B^2
$\mathcal{R}'_2$	IF x_1 in A_1^3 and x_2 in A_2^1 and x_3 in A_3^1 then y in B^1
$\mathcal{R}'_3$	IF x_1 in A_1^2 and x_2 in A_2^2 and x_3 in A_3^2 then y in B^2
$\mathcal{R}'_4$	IF x_1 in A_1^1 and x_2 in A_2^1 and x_3 in A_3^2 then y in B^2
$\mathcal{R}'_5$	IF x_1 in A_1^3 and x_2 in A_2^1 and x_3 in A_3^1 then y in B^2

Fuzzy inference engine combines the fuzzy IF-THEN rules in the fuzzy rule base into a mapping from a fuzzy set $A' \in U$ into a fuzzy set $B' \in V$. In that sense all rules in fuzzy rule base are combined in a single fuzzy set. Namely, the N IF-THEN rules are interpreted as fuzzy relation $\mathcal{Q}_G$ in $U \times V$ defined as:

$$\mathcal{Q}_G = \bigcup_{p=1}^{N} \mathcal{R}_p.$$

We will use Gödel combination to obtain output of the fuzzy inference engine:

$$\mathcal{B}'(y) = \sup_{X \in U} \mathcal{A}'(X) * \mathcal{Q}_G(X, y),$$

where the t-norm $*$ is algebraic product.

5. Conclusion

This paper presents a fuzzy inference engine based on a parameterized algebraic product, together with the corresponding parameterized inference mechanism. Following the methodology introduced in Section 2, a systematic algorithm for implementing the proposed inference engine is developed. The proposed multiplicative model is demonstrated through a hypothetical example, illustrating its applicability and providing a foundation for further investigation and practical implementation in fuzzy reasoning systems.

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References

[1] T. Carvalho, M. Vellasco, J.F. Amaral, *Automatic generation of fuzzy inference systems for multivariate time series forecasting*, Fuzzy Sets and Systems, 470 (2023), Article ID: 108657, 1-19.

[2] D. Dubois, H. Prade, *What are fuzzy rules and how to use them*, Fuzzy Sets Systems, 84 (1996), 169-185.

[3] S. Guillaume, *Designing Fuzzy Inference Systems from Data: An Interpretability-Oriented Review*, IEEE Transactions on Fuzzy Systems, 9 (3) (2001), 426-443.

[4] S.S. Izquierd, L.R. Izquierdo, *Mamdani Fuzzy Systems for Modelling and Simulation: A Critical Assessment*, Journal of Artificial Societies and Social Simulation, 21 (3) 2 (2018), 15 pages.

[5] W. Van Leekwijck, E.E. Kerre, *Defuzzification: criteria and classification*, Fuzzy Sets Systems, 108 (1999), 159-178.

[6] E.H. Mamdani, *Applications of fuzzy algorithm for control a simple dynamic plant*, Proceedings of the Institution of Electrical Engineers, 121 (12) (1974), 585-1588.

[7] E.H. Mamdani, S. Assilian, *An experiment in linguistic synthesis with a fuzzy logic controller*, International Journal Man-Machines Studies, 7 (1) (1975), 1-13.

[8] I. Micić, N. Damljanović, Z. Jančić, *Automated method for designing fuzzy systems*, Facta Universitatis, Series: Mathematics and Informatics, 35 (5) (2020), 1357-1368.

[9] S.K. Pal, D.P. Mandal, *Fuzzy Logic and Approximate Reasoning: An Overview*, IETE Journal of Research, 37 (2015), 548-560.

[10] V. Novak, *Reasoning about mathematical fuzzy logic and its future*, Fuzzy Sets Systems, 192 (2012), 25-44.

[11] V. Novak, *Perception-Based Logical Deduction*, in: B. Reusch (eds), Computational Intelligence, Theory and Applications, Advances in Soft Computing, vol 33. Springer, Berlin, Heidelberg, 2005.

[12] I. Perfiljeva, V. Novak, P. Artiemjew, *Fuzzy Modeling Methods in Processing of Long Time Series*, in: Agnieszka Niemczynowicz, Irina Perfilieva, Lluís M. García-Raffi, and Radosław Kycia (eds), Complex, Hypercomplex and Fuzzy-Valued Neural Networks, New York: Routledge, 2026, pp. 126-143.

[13] W. Pedrycz, *Fuzzy Modelling: Paradigms and Practice*, Kluwer Academic Press, 1996.

[14] T. Takagi, M. Sugeno, *Fuzzy identification of systems and its applications to modeling and control*, IEEE Transactions on Systems, Man, and Cybernetics: Systems, 15 (1) (1985), 116-132.

[15] L.-X. Wang, *A Course In Fuzzy Systems and Control*, Prentice Hall PTR, 1997.

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